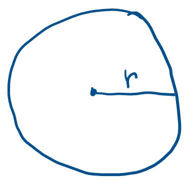
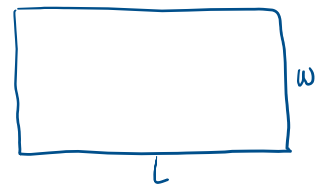
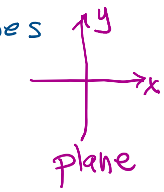
Recall: Finding Standard Areas ← 2-dimensional shapes



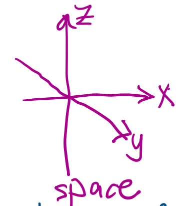
$A_{CIRCLE} = \pi r^2$



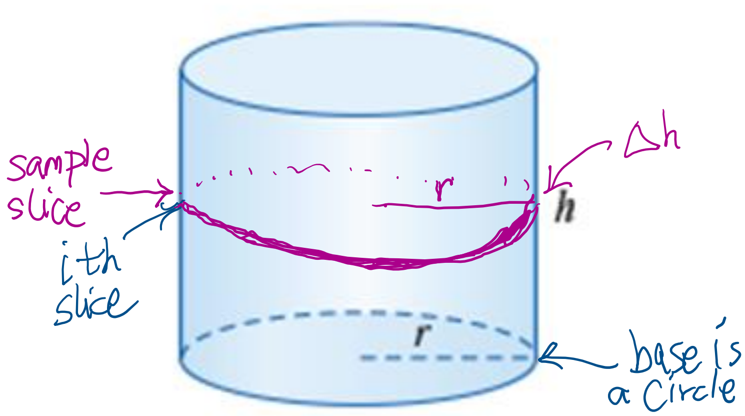
$A_{RECTANGLE} = l \cdot w$



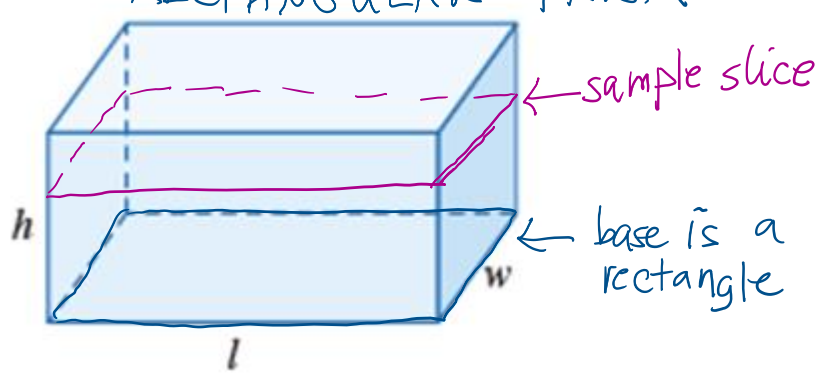
Recall: Finding Corresponding Volumes ← 3-dimensional shapes



CYLINDER



RECTANGULAR PRISM



$V_{CYL} = A_{base} \cdot h$

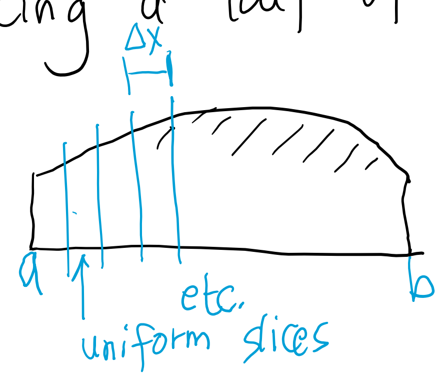
$= \underbrace{\pi r^2}_{A_{SLICE}} \cdot h$

$\lim_{n \to \infty} \left(\sum_{i=1}^{n} \pi r_i^2 \right) \cdot h$

$V = A_{base} \cdot h$

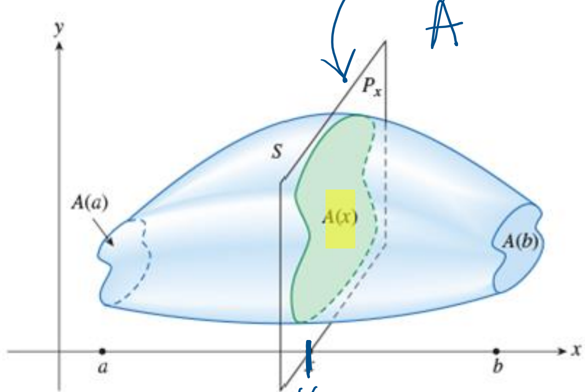
$= lw \cdot h$

ex. slicing a loaf of bread

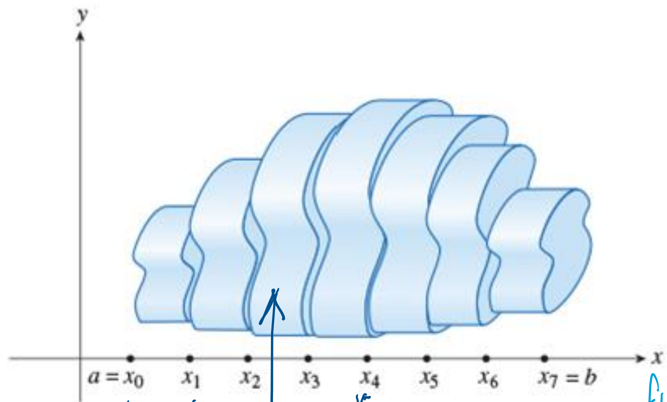


Volume of Uncommon Shapes

slice
A



x_i



x_i

Δx

function $A(x)$

$$V_{SLICE} = A \cdot \Delta x$$

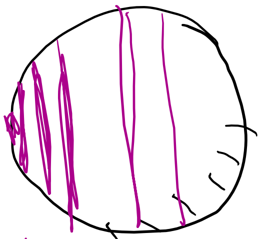
$$V_{LOAF} \approx \sum A_i \cdot \Delta x$$

$$V = \lim_{n \to \infty} \sum_{i=1}^{n} A(x_i) \Delta x$$

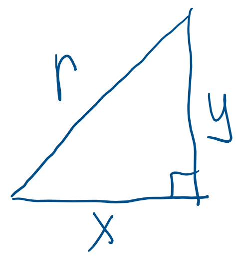
$$V = \int_a^b A(x)\, dx$$

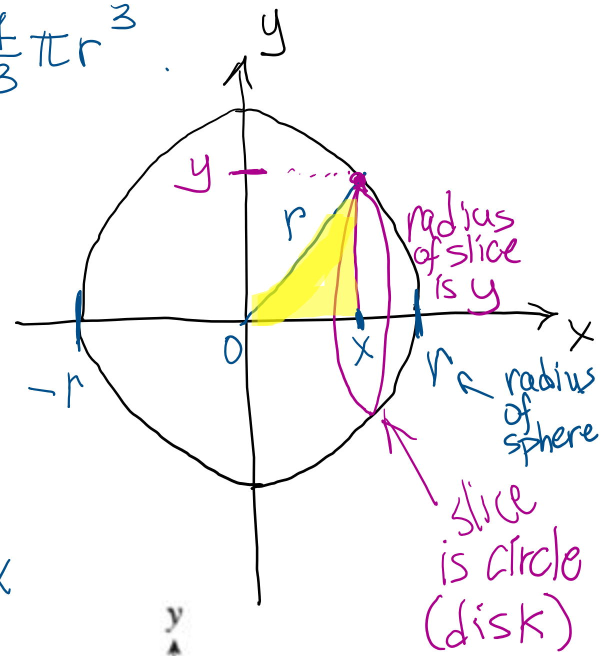
ex. Use integration to show the volume of a sphere with radius, *r*, is $\frac{4}{3}\pi r^3$.

SPHERE



radius of each slice varies in sphere





recall area slice: $A(x)$

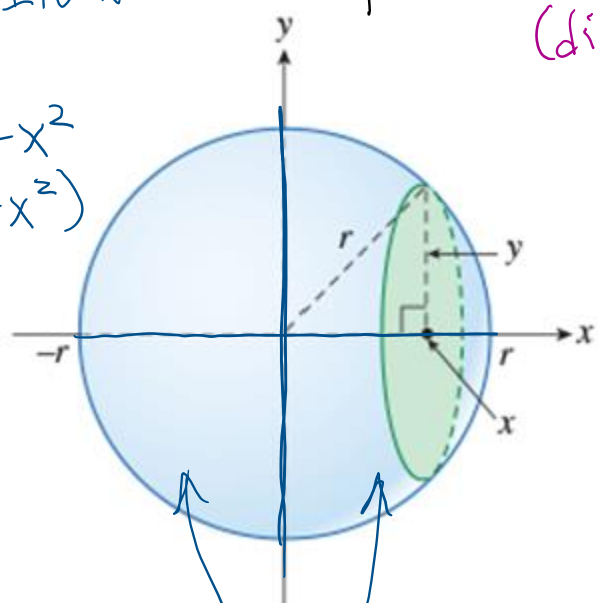
$A_{SLICE} = \pi y^2$

write INTO x

$x^2 + y^2 = r^2$

$y^2 = r^2 - x^2$

$A(x) = \pi(r^2 - x^2)$



Symmetry

$$V_{SPHERE} = \int_{-r}^{r} A_{SLICE}\, dx$$

$$= \pi \int_{-r}^{r} (r^2 - x^2)\, dx$$

treat r as a constant

$$= 2\pi \int_{0}^{r} (r^2 - x^2)\, dx$$

$$= 2\pi \left(r^2 x - \frac{x^3}{3} \right) \Big|_0^r$$

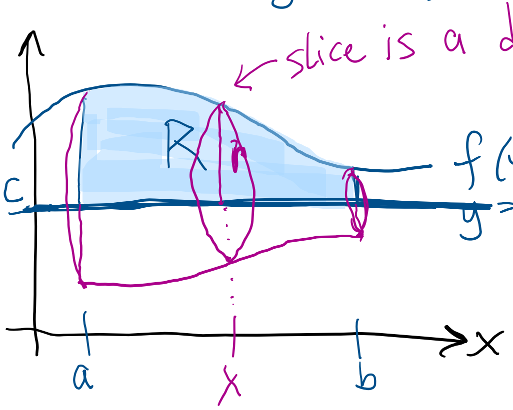
$$= 2\pi \left(r^2 \cdot r - \frac{r^3}{3} - 0 \right)$$

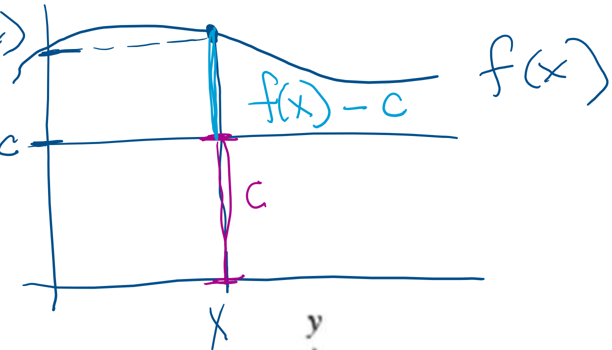
$$= 2\pi \left(\frac{3r^3}{3} - \frac{r^3}{3} \right)$$

$$= 2\pi \cdot \frac{2r^3}{3} = \boxed{\frac{4\pi r^3}{3}}$$

Create a Solid Using the Revolution of a 2-dimensional Region

construct region, R, under curve, $f(x)$, and line $y=c$ then revolve R about $y=c$

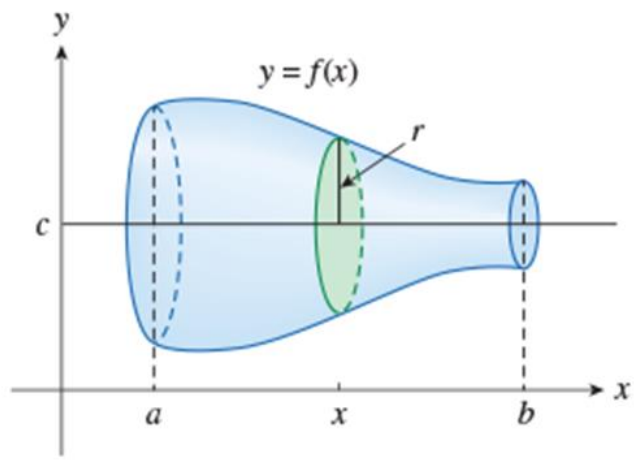




r of disk is $f(x)-c$

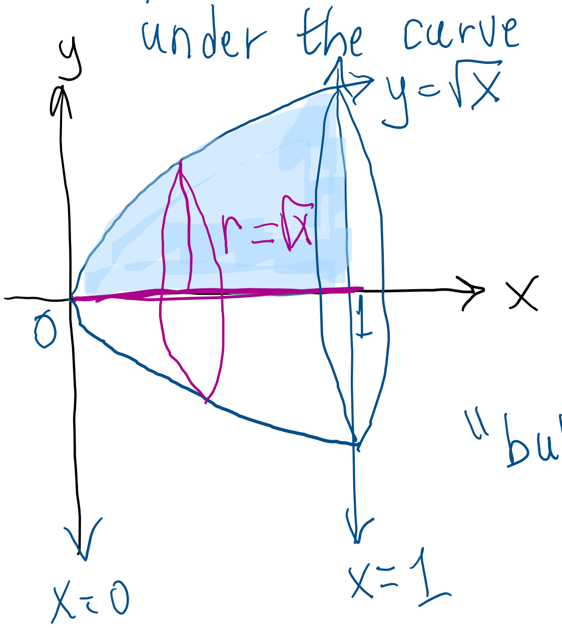
$$A_{DISK} = \pi r^2$$
$$= \pi (f(x)-c)^2$$

slice



Find Volume Using The Disk Method

ex. find volume of a solid obtained by rotating the region, under the curve $y=\sqrt{x}$ from $x=0$ to $x=1$ about x-axis

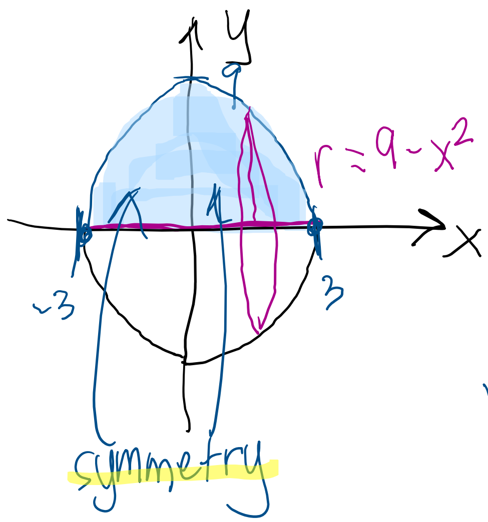


"bullet"

$$A_{SLICE} = \pi r^2$$
$$= \pi \sqrt{x}^2$$
$$= \pi x$$

$$V = \pi \int_0^1 x \, dx$$
$$= \pi \frac{x^2}{2}\Big|_0^1$$
$$= \frac{\pi}{2} x^2 \Big|_0^1$$
$$= \frac{\pi}{2}(1-0) = \boxed{\frac{\pi}{2}}$$

ex. Find the volume of the solid obtained by rotating the region bound by $y = 9 - x^2$ and x-axis around x-axis.

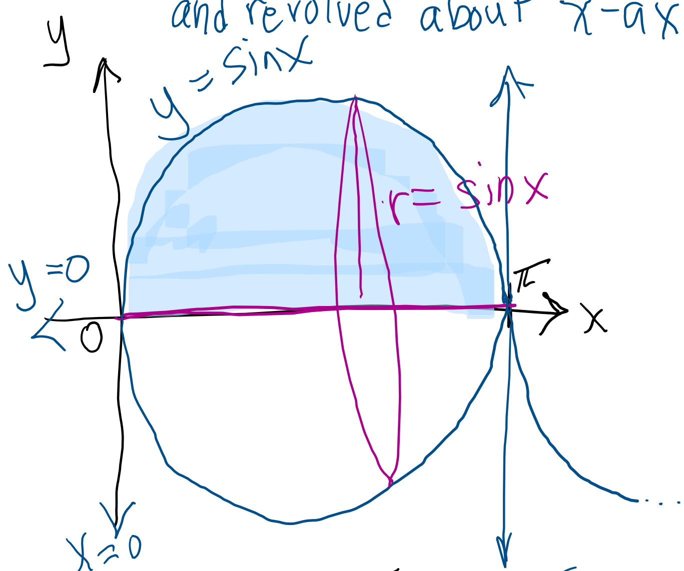


integration bounds are x-intercepts:

$$9 - x^2 = 0$$
$$(3-x)(3+x) = 0$$
$$x = \pm 3$$

$$V = \pi \int_{-3}^{3} (\underbrace{9 - x^2}_{r})^2 \, dx$$
$$= 2\pi \int_0^3 (9 - x^2)(9 - x^2) \, dx$$
$$= 2\pi \int_0^3 (81 - 18x^2 + x^4) \, dx$$
$$= 2\pi \left(81x - \cancel{18}^{\,6} \frac{x^3}{\cancel{3}} + \frac{x^5}{5} \right) \Big|_0^3$$
$$= 2\pi \left(81(3) - 6(27) + \frac{3^5}{5} - 0 \right) = \boxed{\frac{524\pi}{5}}$$

ex. Find the volume of the solid obtained by rotating the region bound by $y = \sin x$, $y = 0$, $x = 0$, $x = \pi$ and revolved about x-axis.



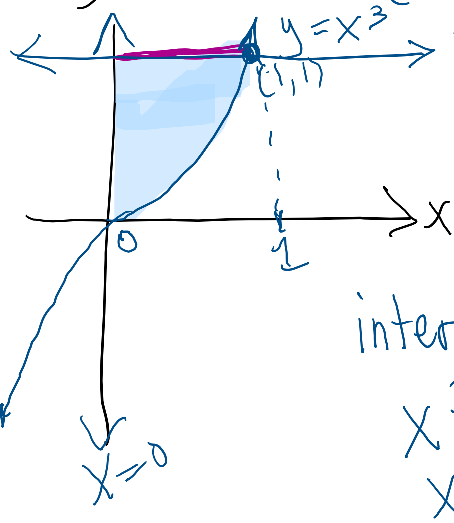
$$V = \pi \int_0^{\pi} \underbrace{\sin^2 x}_{} \, dx$$

$\cos 2x = 1 - 2\sin^2 x$

$\frac{1}{2}(1 - \cos 2x) = \sin^2 x$

$$= \frac{1}{2}\pi \int_0^{\pi} (1 - \cos 2x) \, dx$$
$$= \frac{\pi}{2} \left(x - \frac{1}{2} \sin 2x \right) \Big|_0^{\pi}$$
$$= \frac{\pi}{2} \left(\pi - \frac{1}{2} \cancelto{0}{\sin 2\pi} - \left(0 - \frac{1}{2} \cancelto{0}{\sin 0} \right) \right)$$
$$= \boxed{\frac{\pi^2}{2}}$$

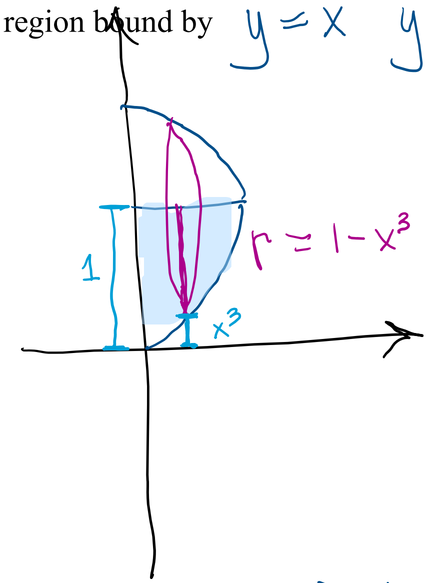
ex. Find the volume of the solid obtained by rotating the region bound by $y = x^3$, $y = 1$, $x = 0$ about the line $y = 1$.



intersection:

$x^3 = 1$

$x = 1$



$$V = \pi \int_0^1 (1 - x^3)^2 \, dx$$